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TIME AND FREQUENCY DOMAIN ADAPTIVE ALGORITHMS FOR APPROXIMATING--ETC(U)

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TIME AND FREQUENCY DOMAIN ADAPTIVE ALGORITHMS FOR APPROXIMATING THE FILTERING OPERATIONS IN AN OPTIMUM MULTIBEAM SYSTEM

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INTRODUCTION

The purpose of this memorandum is to derive several adaptive algorithms for approximately realizing the matrix filtering operation prior to beamforming which is utilized by a wide class of "optimum" multibeam detection and estimation systems. The "optimum" array systems that are referred to here generally assume a plane wave signal or some known linear transformation of a plane wave signal or some known linear transformation of a plane wave signal, stationary input processes, and an observation interval which is long compared to the maximum interval of significant correlation of the input processes. A wide variety of estimation and detection criteria have been investigated* under these assumptions; the filtering and beamforming operations have been shown to all fit into the compact cannonical form illustrated in Fig. 1.

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*See Van Trees [1], Nuttall and Hyde [2], and Cox [3].

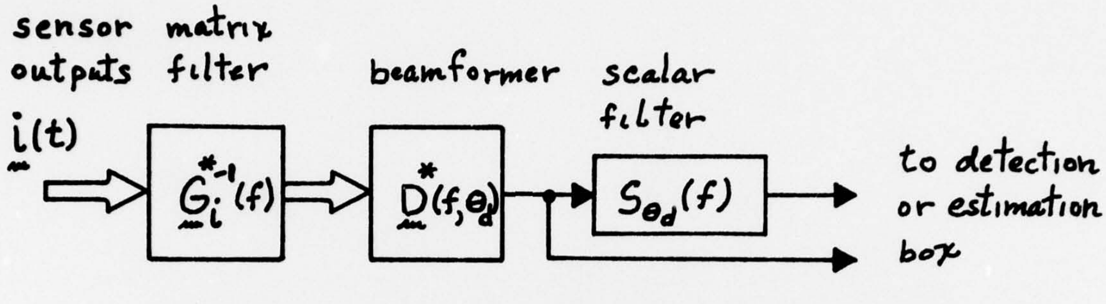


Fig. 1 - A cannonical form of the array combining and filtering operations for the reception of a plane wave signal.

In Fig. 1, the power spectral matrix of the K element sensor output vector $\underset{\sim}{i}(t)$ is taken to be $\underset{\sim}{G}_i(f)$ and the single look angle illustrated is denoted by θ_d . (In general θ_d may be taken to be an ordered pair of angles.) Since the matrix filtering operation does not depend on the particular look angle, this formulation of the optimum processor is particularly amenable to the simultaneous realization of a large number of beams. The implementation of multiple look directions would involve only parallel delay-sum operations in the beamformer box and the specification of the appropriate scalar filters and subsequent circuitry for each look direction. Depending on the particular processor, the scalar filters may or may not functionally depend on the look angles and/or the spectral matrix of the data.

The adaptive specification of a matrix filter approximating the $\underset{\sim}{G}_i^{*-1}(f)$ filter either as a matrix of K^2 complex weights of each frequency of interest, or as a set of K^2 L-tap delay line filters, will be the main topic under consideration in this memorandum. For some processors, the specification of the scalar filters may also require an adaptive technique, but this problem will not be considered in any detail. When the scalar filters are noise field dependent, as in the maximum likelihood estimation case, their frequency transfer functions involve operations on the $\underset{\sim}{G}_i^{*-1}(f)$ matrix and they can be specified directly once the matrix filter is designed.

In Section 1 to follow, basic filtering theory is reviewed, leading to the derivation of an adaptive frequency domain algorithm and an adaptive time domain algorithm for discrete parameter matrix filters in Section 2 which approximately realize a given input-output cross correlation (CC) or cross

spectral (CS) property. Following in Section 3, these algorithms are examined and applied to the realization of the $\underset{\sim}{G}_i^{*-1}(f)$ matrix filter, and in Section 4 some simulation results are given which illustrate several aspects of the performance of the time domain adaptive filter system against plane wave interferences.

1. ADAPTIVE DESIGN OF FILTERS WITH SPECIFIED INPUT-OUTPUT CROSSCORRELATION PROPERTIES

In this section, the input-output conventions are established for matrix filters, both in the time domain and the frequency domain. Then, the structure of filters which result in a given input-output CC or CS is developed. This class of filters is linked to the general class of unrealizable Wiener filters by noticing that a Wiener filter which minimizes the m.s. error between some desired signal and the input such that the CC or CS between the two is specified actually results in that CC or CS between input and output.

Practical realizations of filter structures, i.e., those having discrete time or frequency domain weights, are then developed as approximations to their continuous parameter counterparts. Conditions for accurate representation of the continuous filters are reviewed for the time and frequency domain structures, and the constrained filter structures which approximately realize a specified CC or CS are given.

1.1 FUNDAMENTAL CONCEPTS

Following the notations in [2] (sec. 2) we adopt the convention that an N input, M output (time invariant) linear filtering operation is given in the time domain by

$$o_k(t) = \sum_{j=1}^{N} \int d\tau\, h_{kj}(\tau)\, i_j(t-\tau)\,, \quad 1 \le k \le M, \tag{1}$$

and in the frequency domain by

$$Y_k(f) = \sum_{j=1}^{N} H_{kj}(f) X_j(f)\,. \tag{2}$$

Forming the matrices

$$\underset{\sim}{H}_k^T(f) \triangleq [\, H_{k1}(f), \ldots, H_{kN}(f) \,], \quad 1 \le k \le M,$$

$$\underset{\sim}{X}^T \triangleq [X_1(f), \ldots, X_N(f)],$$

Equation (2) can be written

$$Y_k(f) = \underset{\sim}{H}_k^T(f)\underset{\sim}{X}(f), \quad 1 \le k \le M. \tag{3}$$

Further specifying

$$\underset{\sim}{Y}^T(f) \triangleq [Y_1(f), \ldots, Y_M(f)],$$

and

$$\underset{\sim}{H}(f) \triangleq [\underset{\sim}{H}_1(f), \ldots, \underset{\sim}{H}_M(f)],$$

Equation (3) can be compactly written

$$\underset{\sim}{Y}(f) = \underset{\sim}{H}^T(f)\underset{\sim}{X}(f). \tag{4}$$

In the time domain

$$\underset{\sim}{o}(t) = \int d\tau\, \underset{\sim}{h}^T(\tau)\underset{\sim}{i}(t-\tau). \tag{5}$$

Define the input-output CC as

$$\underset{\sim}{R}_{io}(\tau) \triangleq \overline{\underset{\sim}{i}(t)\underset{\sim}{o}^H(t-\tau)} = \int du\, \underset{\sim}{R}_i(\tau+u)\underset{\sim}{h}^*(u) \tag{6}$$

where $\underset{\sim}{R}_i(\tau) \triangleq \overline{\underset{\sim}{i}(t)\underset{\sim}{i}^H(t-\tau)}$ and $(\cdot)^H$ denotes a transpose-complex conjugate, i.e., $(\cdot)^{*T} \equiv (\cdot)^H$. Then the input-output CS is

$$\underset{\sim}{G}_{io}(f) = \int d\tau\, e^{-2\pi i f\tau}\underset{\sim}{R}_{io}(\tau) = \underset{\sim}{G}_i(f)\underset{\sim}{H}^*(f) \tag{7}$$

where

$$\underset{\sim}{G}_i(f) \triangleq \int d\tau\, e^{-2\pi i f\tau}\underset{\sim}{R}_i(\tau).$$

The frequency domain transfer function of a matrix filter which results in a prescribed input-output CS, $\underset{\sim}{G}_d(f)$, is immediately seen from Eq. (7) to be

$$\underset{\sim}{H}^*(f) = \underset{\sim}{G}_i^{-1}(f)\underset{\sim}{G}_d(f). \tag{8}$$

Hence, if $\underset{\sim}{G}_d(f)$ is the desired CS then the structure of the filter which realizes it is completely specified with additional knowledge of the matrix inverse of the input power spectrum. In particular, if the desired CS is white and N equals M, then $\underset{\sim}{G}_d(f)$ is just the identity matrix, and

$$\underset{\sim}{H}(f) = \underset{\sim}{G}_i^{*-1}(f). \tag{9}$$

It is interesting to note that the well-known class of Wiener filters realize a specified input-output CC or CS corresponding to the CC or CS between the desired output and the input. Define the m.s. error matrix at the filter output by

$$\underline{\epsilon}^2 = \overline{[\underline{d}(t) - \underline{o}(t)][\underline{d}(t) - \underline{o}(t)]^H}. \tag{10}$$

This m.s. error matrix can be expanded in a manner similar to [2] (Sec. 3.2) into a perfect square and a remainder,

$$\begin{aligned}
\underline{\epsilon}^2 &= \underline{R}_d(0) - \int d\tau\, \underline{h}^T(\tau)\underline{R}_{di}^H(\tau) - \int d\tau\, \underline{R}_{di}(\tau)\underline{h}^*(\tau) \\
&\qquad + \iint d\tau_1\, d\tau_2\, \underline{h}^T(\tau_1)\underline{R}_i(\tau_2-\tau_1)\underline{h}^*(\tau_2), \\
&= \underline{R}_d(0) - \int df\, \underline{H}^T(f)\underline{G}_{di}^H(f) - \int df\, \underline{G}_{di}(f)\underline{H}^*(f) \\
&\qquad + \int df\, \underline{H}^T(f)\underline{G}_i(f)\underline{H}^*(f), \\
&= \underline{R}_d(0) - \int df\, \underline{G}_{di}^H(f)\underline{G}_i^{-1}(f)\underline{G}_{di}(f) \\
&\qquad + \int df\, [\underline{H}^*(f) - \underline{G}_i^{-1}(f)\underline{G}_{di}(f)]^H \underline{G}_i(f)[\underline{H}^*(f) - \underline{G}_i^{-1}(f)\underline{G}_{di}(f)].
\end{aligned} \tag{11}$$

The elements of the error matrix are minimized when the last term in Eq. (11) is zero, resulting in a familiar expression for $\underline{H}(f)$,

$$\underline{H}^*(f) = \underline{G}_i^{-1}(f)\underline{G}_{di}(f). \tag{12}$$

Hence, if $\underline{d}(t)$ is any signal vector such that $\underline{G}_{di}(f)$ is the desired input-output CS the Wiener filter, Eq. (12), is identical to Eq. (8) and the Wiener filter realizes that input-output CS. This property of Wiener filters will be exploited in Section 2 to derive adaptive filter algorithm to realize a specified input-output CC or CS.

1.2 PRACTICAL FILTER STRUCTURES

A filter structure that is amenable to adjustment of its parameters generally requires that its time or frequency domain function be sampled in

some sense, i.e., that its adjustment be relegated to a few easily controlled coefficients. Two such filter models we wish to consider here are

$$\underset{\sim}{h}(\tau) \cong \sum_{\ell=0}^{L-1} \underset{\sim}{a}_\ell \, r_\ell(\tau+\tau_0), \tag{13}$$

$$\underset{\sim}{H}(f) \cong \sum_{\ell=0}^{L-1} \underset{\sim}{A}_\ell \, R_\ell(f-f_0) \tag{14}$$

where, in particular, the basis functions $\{r_\ell(\tau+\tau_0)\}$ are just uniformly delayed impulses $\{\delta(\tau-\ell\Delta+\tau_0)\}$ and the set $\{R_\ell(f-f_0)\}$ are $\{\delta(f-\ell\gamma-f_0)\}$

The coefficients $\{\underset{\sim}{a}_\ell\}$ are given by

$$\begin{aligned} \underset{\sim}{a}_\ell &= \int d\tau \, \delta(\tau+\tau_0-\Delta\ell)\, \underset{\sim}{h}(\tau) \\ &= \underset{\sim}{h}(\Delta\ell-\tau_0), \end{aligned} \tag{15}$$

and

$$\begin{aligned} \underset{\sim}{A}_\ell &= \int df \, \delta(f-f_0-\gamma\ell)\, \underset{\sim}{H}(f) \\ &= \underset{\sim}{H}(\gamma_\ell+f_0). \end{aligned} \tag{16}$$

If the maximum extent in τ in which $\underset{\sim}{h}(\tau)$ has significant power is denoted denoted by T and the minimum interval in which $\underset{\sim}{h}(\tau)$ "wiggles" or varies significantly is denoted by D , then the conditions under which Eq. (13) is a good approximation are $\Delta L > T$ and $\Delta \ll D$. The condition under which Eq. (14) is a good approximation for an $\underset{\sim}{H}(f)$ which has extent F and "wiggles" W are $\gamma L > F$ and $\gamma \ll W$. These frequency and time domain conditions can be related by noting that $T \sim 1/W$ and $D \sim 1/2F$.

The input-output relations of these TDL constrained filters are, in the time domain,

$$o_k(t) = \sum_{\ell=0}^{L-1} \underset{\sim}{h}_k^T(\ell\Delta)\, \underset{\sim}{i}(t-\ell\Delta+\tau_0) \;, 1 \le k \le M, \tag{17}$$

$$= \underset{\sim}{W}_k^T \underset{\sim}{I}(t), \tag{18}$$

where the NL dimensional vector $\underset{\sim}{W}_k$ is defined by

$$\underset{\sim}{W}_k^T \triangleq \left[\underset{\sim}{h}_k^T(0), \ldots, \underset{\sim}{h}_k^T((L-1)\Delta) \right], \tag{19}$$

and

$$\underset{\sim}{I}^T(t) \triangleq \left[\underset{\sim}{i}^T(t+\tau_0), \ldots, \underset{\sim}{i}^T(t-(L-1)\Delta+\tau_0) \right] \tag{20}$$

Then

$$\underset{\sim}{o}(t) = \underset{\sim}{W}^T \underset{\sim}{I}(t) \tag{21}$$

where $\underset{\sim}{W}$ is a NL x M matrix. In the frequency domain

$$\underset{\sim}{Y}(f_\ell) = \underset{\sim}{H}^T(f_\ell) \underset{\sim}{X}(f_\ell), \quad \ell = 1, \ldots, L. \tag{22}$$

Hence, the general input-output expression (14) needs only to be specified at the various frequencies of interest.

The input-output CC for a TDL filter is from Eqs. (6) and (21)

$$\underset{\sim}{R}_{io}(\tau) = \overline{\underset{\sim}{i}(t)\underset{\sim}{I}^H(t-\tau)}\,\underset{\sim}{W}^* = \underset{\sim}{R}_i'(\tau)\,\underset{\sim}{W}^* \tag{23}$$

where $\underset{\sim}{R}_i'(\tau)$ is a N x NL matrix $\underset{\sim}{R}_i'(\tau) \triangleq \overline{\underset{\sim}{i}(t)\underset{\sim}{I}^H(t-\tau)}$.

Letting τ take on only the L values $\{(\Delta m-\tau_0)\}, 0 \le m \le L-1$, and defining the NL x M matrix $\underset{\sim}{R}_{io}'(\tau_0)$,

$$\underset{\sim}{R}_{io}'^T(\tau_0) = \left[\underset{\sim}{R}_{io}^T(\tau_0), \ldots, \underset{\sim}{R}_{io}^T(\tau_0-(L-1)\Delta) \right] \tag{24}$$

Equation (23) can be written

$$\underset{\sim}{R}_{io}'(\tau_0) = \underset{\sim}{R}_i\,\underset{\sim}{W}^*, \tag{25}$$

where $\underset{\sim}{R}_i$ is a NL x NL matrix of values, $\underset{\sim}{R}_i \triangleq \overline{\underset{\sim}{I}(t)\underset{\sim}{I}^H(t)}$. Hence, the

tap weight maxtrix $\underset{\sim}{W}$ which realizes a prescribed CC matrix $\underset{\sim}{R}_d(\tau_o)$ is given by

$$\underset{\sim}{W} = \underset{\sim}{R}_i^{-1} \underset{\sim}{R}_d(\tau_o) \tag{26}$$

where the $(\cdot)^*$ has been dropped since the weights are taken to be real.

In the frequency domain the constrained filter which gives a prescribed CS at the various frequencies of interest can be trivially extended from Eq. (8).

$$\underset{\sim}{H}(f_\ell) = \underset{\sim}{G}_i^{*-1}(f_\ell)\, \underset{\sim}{G}_d(f_\ell) \,, \quad 1 \le \ell \le L \,. \tag{27}$$

The sampled frequency domain transfer function of the Wiener filter which realizes a prescribed CS is also given by Eq. (27).

2. ADAPTIVE DESIGN OF FREQUENCY SAMPLED AND TAPPED DELAY LINE FILTERS FOR A SPECIFIED CC OR CS

The adaptive algorithms developed in this section are generalizations of a time domain beamformer algorithm investigated by Griffiths [4] (Sec. IV.C) for obtaining the Wiener (LMS) weights to optimally estimate a plane wave signal. Although the class of matrix filters which give a prescribed input-output CC or CS is certainly much larger than the class of filters for LMS beamforming, the basic philosophy of Griffith's algorithm is applicable.

The philosophy of the Griffiths algorithm can be stated in words as follows: Starting from an arbitrary matrix of weights, recursively update the weights in succeeding time instants by adding to the old matrix a scaled version of a matrix comprised of the difference between the desired CC or CS matrix and an instantaneous or short-time estimate of the actual input-output CC or CS. In the LMS beamformer application Griffiths* showed that the sequence of weights obtained

*See [4] Sec. V.

by such an algorithm converged in the mean to the Wiener weights and that the variance of the steady-state weights was bounded and proportional to the scale factor in the update equation.

We will generalize this approach to the adaptive specification of matrix filters which realize a prescribed CC or CS matrix by presenting two general algorithms--one for the frequency domain weights and one for TDL weights. In each case, the convergence of the mean will be investigated and an upper bound on the variance of the steady-state weights will be given. In each case the input processes are taken to be stationary and to possess the Gaussian fourth movement separability property.

2.1 A FREQUENCY DOMAIN ADAPTIVE ALGORITHM FOR REALIZING A FILTER WITH PRESCRIBED CS PROPERTIES

Considering first the frequency domain case, let a record of the senor data vector by Fourier transformed after multiplication by some window $b(t+T_n)$ with absolute time reference T_n, i.e.,

$$\underset{\sim}{X}_n(f) \triangleq \int dt\, e^{-2\pi i f t} b(t+T_n)\, \underset{\sim}{i}(t) \tag{28}$$

where

$$b(t+T_n) \cong 0\,, \quad T_n - T \geq t\,, \; T_n + T \leq t\,.$$

As in Section 1.1 $\underset{\sim}{i}(t)$ is an N vector.

The power spectrum of the transform $\underset{\sim}{X}_n(f)$ is

$$\underset{\sim}{G}'_i(f) \triangleq \overline{\underset{\sim}{X}_n(f)\underset{\sim}{X}_n^H(f)} = \iint dt_1\, dt_2\, e^{2\pi i f(t_2-t_1)} b(T_n+t_1) b(T_n-t_2)\, \overline{\underset{\sim}{i}(t_1)\, \underset{\sim}{i}^H(t_2)}$$

$$= \int d\tau\, e^{-2\pi i f \tau} R_b(\tau) \underset{\sim}{R}_i(\tau) \tag{29}$$

$$= \int d\lambda\, G_b(f-\lambda)\, \underset{\sim}{G}_i(\lambda)\,, \tag{30}$$

where $R_b(\tau)$ is the (scalar) autocorrelation of the window function and $G_b(f)$

is the spectrum. If $G_i(f)$ is of small non-zero extent compared to the wiggles in the approximation to the true spectrum is very good, and,

$$G_i'(f) \cong G_i(f) . \tag{31}$$

Now, with Eq. (4) defining the output vector $Y_n(f)$ of the filter $H_n(f)$, consider the following recursion relation,

$$H^*_{n+1}(f) = H^*_n(f) + \mu \left[G_d(f) - X_n(f) Y_n^H(f) \right] . \tag{32}$$

Here, $H_n(f)$ is the matrix of complex weights at frequency f at the n-th update time, $G_d(f)$ is the desired input-output CS at frequency f, $X_n(f)Y_n^H(f)$ is the instantaneous CS computed from the data, and μ is a small constant to be determined.

The matrix of complex weights $H_n(f)$ can be shown to converge in the mean to the solution

$$\lim_{n \to \infty} \overline{H^*_n(f)} \triangleq M^*_H(f) = G_i'^{-1} G_d(f) \tag{33}$$

as follows:*

Let the weights H_n be updated at time instants $\{t_i\}$ taken far enough apart so that the windowed transforms $\{X_n\}$ are independent. Then taking the means of Eq. (32) and using Eq. (4)

$$\begin{aligned}
\overline{H^*_{n+1}(f)} \triangleq M^*_{H_{n+1}}(f) &= \overline{H^*_n(f)} + \mu \left[G_d(f) - \overline{X_n(f) X_n^H(f)}\ \overline{H^*_n(f)} \right] \\
&= M^*_{H_n}(f) + \mu G_d(f) - \mu G_i'(f) M^*_{H_n}(f) \\
&= \left(I - \mu G_i'(f) \right) M^*_{H_n}(f) + \mu G_d(f) \\
&= \left[I - \mu G_i'(f) \right]^{n+1} H^*_0(f) + \sum_{i=0}^{n} \left(I - \mu G_i'(f) \right)^i \mu G_d(f) .
\end{aligned} \tag{34}$$

*This proof generally follows Griffiths' proof of the convergence of the LMS weights [4] Sec. V. The assumption that the data be independent at succeeding update instants is not necessary, but greatly simplifies the proof. See Daniell [5].

Utilizing the identity

$$\sum_{i=0}^{n-1} \underset{\sim}{Z}^i \equiv \left[\underset{\sim}{I} - \underset{\sim}{Z}^n\right]\left[\underset{\sim}{I} - \underset{\sim}{Z}\right]^{-1}$$

Eq. (34) becomes

$$\underset{\sim}{M}^*_{H_n}(f) = \left[\underset{\sim}{I} - \mu \underset{\sim}{G}_i'(f)\right]^n \left[\underset{\sim}{H}_o^*(f) - \underset{\sim}{G}_i'^{-1}(f)\underset{\sim}{G}_d(f)\right] + \underset{\sim}{G}_i'^{-1}(f)\,\underset{\sim}{G}_d(f)\,. \tag{35}$$

Now

$$\lim_{n \to \infty} \left[\underset{\sim}{I} - \mu \underset{\sim}{G}_i'(f)\right]^n = 0$$

if

$$0 < \mu < 2/\rho_{max}(f)\,, \tag{36}$$

where $\rho_{max}(f)$ is the maximum eigenvalue of $\underset{\sim}{G}'(f)$. Hence, if Eq. (36) holds,

$$\lim_{n \to \infty} \underset{\sim}{M}^*_{H_n}(f) = \underset{\sim}{M}^*_H(f) = \underset{\sim}{G}_i'^{-1}(f)\,\underset{\sim}{G}_d(f)\,. \tag{37}$$

Also,

$$\underset{\sim}{M}^*_H(f) \cong \underset{\sim}{G}_i^{-1}(f)\underset{\sim}{G}_d(f) \tag{38}$$

when the window function for the data transform is properly chosen.

An upper bound on the variance of the weight reached in the steady state via Eq. (32) can also be derived. Rather than present this cumbersome derivation which is similar to Griffiths' ([4], Sec. V.2) we shall refer the reader to that work for the derivation and simply give the result.

Following Eq. (2), let $\underset{\sim}{H}_j(f)$ and $\underset{\sim}{H}_k(f)$ be the j-th and k-th columns from the steady state weight matrix. Defining

$$\underset{\sim}{C}_{jk}(f) \triangleq \overline{\underset{\sim}{H}_j(f)\underset{\sim}{H}_k^H(f)} - \underset{\sim}{M}_{H_j}(f)\,\underset{\sim}{M}^H_{H_k}(f)\,, \tag{39}$$

use as a weight variance measure $tr\{C_{ij}\}$. Then, similar to Griffiths' result,

$$tr\{C_{jk}(f)\} \le \frac{\mu}{\rho_{min}(f)}\left[\frac{\gamma(f)+\rho_{max}(f)}{2-\mu\gamma(f)-\mu\rho_{max}(f)}\right] G_{d_j}^H(f)\left[G_i'^{-1}(f)-I\right]G_{d_k}(f), \quad (40)$$

for $\rho_{min}(f)$ the minimum eigenvalue of $G_i'(f)$, $\rho_{max}(f)$ the maximum eigenvalue of $G_i'(f)$ and $\gamma(f) \triangleq tr\{G_i'(f)\}$. For small μ Eq. (40) can be seen to be proportional to μ. Hence, the $tr\{C_{jk}(f)\}$ can be made arbitrarily small by choice of μ for every j and k if $G_i'^{-1}(f)$ is finite.

2.2 AN ADAPTIVE ALGORITHM FOR REALIZING A TDL MATRIX FILTER WITH PRESCRIBED CC PROPERTIES

The TDL matrix filter W which realizes a prescribed NL x M CC matrix $R_d(\tau_0)$ is from Eq. (26),

$$W = R_i^{-1} R_d(\tau_0) \quad (41)$$

Let an arbitrary set of values be be taken on by $W(t_0)$ and let W be updated at succeeding time instants $t_n, t_{n+1}, \cdots$ by the recursion relation

$$W(t_{n+1}) = W(t_n) + \mu\left[R_d(\tau_0) - I(t_n)\, o^H(t_n)\right], \quad (42)$$

where μ is a small constant yet to be determined.

We claim that, as with the previous algorithm, the W matrix converges in the mean to the desired solution, Eq. (41), for properly chosen μ, and that the variance of the steady state weight is bounded and proportional to μ for μ small. The proof of the convergence in mean follows analogously to that of the previous algorithm and will be presented in only a shortened form.

Taking the mean of both sides of Eq. (42) assume that the data vector is independent during succeeding update instants. Then,

$$\overline{W(t_{n+1})} \triangleq M_W(t_{n+1}) = M_W(t_n) + \mu R_d(\tau_0) - \mu\, \overline{I(t_n) I^H(t_n)}\, M_W^*(t_n)$$

$$= \underset{\sim}{M}_W(t_n) + \mu \underset{\sim}{R}_d(\tau_o) - \mu \underset{\sim}{R}_i \underset{\sim}{M}_W(t_n)$$

$$= [\underset{\sim}{I} - \mu \underset{\sim}{R}_i] \underset{\sim}{M}_W(t_n) + \mu \underset{\sim}{R}_d(\tau_o) \, . \qquad (43)$$

Continuing,

$$\underset{\sim}{M}_W(t_n) = [\underset{\sim}{I} - \mu \underset{\sim}{R}_i]^n [\underset{\sim}{W}(t_o) - \underset{\sim}{R}_i^{-1} \underset{\sim}{R}_d(\tau_o)] + \underset{\sim}{R}_i^{-1} \underset{\sim}{R}_d(\tau_o) \, ,$$

and in the limit

$$\lim_{n \to \infty} \underset{\sim}{M}_W(t_n) \triangleq \underset{\sim}{M}_W = \underset{\sim}{R}_i^{-1} \underset{\sim}{R}_d(\tau_o) \qquad (44)$$

where

$$0 < \mu < 2/\rho_{max} \quad ,$$

where ρ_{max} is the maximum eigenvalue of $\underset{\sim}{R}_i$.

The variance of the steady state weights are derived in a fashion exactly analogous to Griffiths' development ([4], Sec V.C) and will not be repeated here.

3. ADAPTIVE ALGORITHMS FOR REALIZING MATRIX FILTERS TO APPROXIMATE THE $\underset{\sim}{G}_i^{*-1}(f)$ MATRIX FILTER

The algorithms (Eqs. (32) and (42)) can be immediately applied to the problem of adaptively specifying the $\underset{\sim}{G}_i^{*-1}(f)$ matrix filter in the optimum multibeam processor. Considering the frequency domain algorithm first, consider the adapting filter system illustrated in Fig. 2.

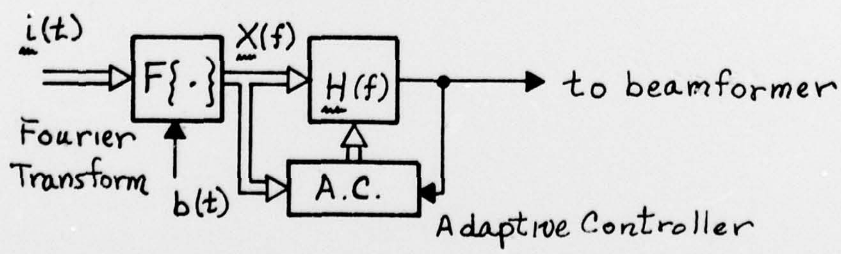


Fig. 2 – A Frequency Domain System

If a set of frequencies $f_1, f_1+\Delta f, \ldots$ is of interest, then the duration of the window function $b(t)$ must be $T > 1/\Delta f$ in order that the data essentially be independent from frequency to frequency. Related to this problem is the spectral smearing of $\underset{\sim}{G}_i(f)$ by the power spectrum of $b(t)$. In order to accurately reflect changes in $\underset{\sim}{G}_i(f)$ of the order of Δf, T must again satisfy $T > 1/\Delta f$. On the other hand, constraints on the size of the Fourier transforming hardware necessarily limit the length of the window function. Also, since only one adaptation per block transform can be made the speed of adaptation additionally constrains the length of $b(t)$. A high adaptation rate would involve repeated transforms of overlapping blocks and would require extremely high speed transforms.

Given these practical considerations let the dimensionality of $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ be equal in keeping with the system in Fig. 1 and let the prescribed CS matrix in the algorithm (Eq. (32)) be

$$\underset{\sim}{G}_d(f) = \underset{\sim}{I}, \qquad f = f_1, f_1+\Delta f, \ldots \tag{45}$$

Then the adaptive controller updates $\underset{\sim}{H}_n(f)$, for the transformed block $\underset{\sim}{X}_n(f)$, by

$$\underset{\sim}{H}^*_{n+1}(f) = \underset{\sim}{H}^*_n(f) + \mu\left[\underset{\sim}{I} - \underset{\sim}{X}_n(f)\underset{\sim}{Y}_n^H(f)\right], \tag{46}$$

and the weights converge in the mean to steady state weights ,

$$\underset{\sim}{H}(f) = \underset{\sim}{G}_i^{*\prime -1}(f) \cong \underset{\sim}{G}_i^{*-1}(f). \tag{47}$$

Time Domain TDL weights can be used to approximate the impulse response structure of the $\underset{\sim}{G}_i^{*-1}(f)$ matrix in a given frequency band by choosing the TDL lengths to span the lengths of the component impulse responses and the tap delay increments less than the widths of the smallest significant wiggles in component impulse responses. Since the desired CS matrix is an identity, the corresponding CC is a matrix of component cross-correlations which are zero for the off diagonal elements and impulse-like for the diagonal elements. That is, for L taps with Δ spacing and N inputs and outputs, $\underset{\sim}{R}_d(\tau_0)$ is a NL x N matrix

$$\underset{\sim}{R}_d(\tau_0) = k\left[\underset{\sim}{0}, \ldots, \underset{\sim}{I}, \ldots, \underset{\sim}{0}\right]^T$$

where $\underset{\sim}{I}$ is a N x N identity matrix, $\underset{\sim}{0}$ is N x N null matrix and k is some

scaling factor. The block of unity values occurs at the tap closest to delay τ_0.

The algorithm for determining the NL x N weight matrix $\underset{\sim}{W}$ is, from Eq. (42)

$$\underset{\sim}{W}(t_{n+1}) = \underset{\sim}{W}(t_n) + \mu \left[\underset{\sim}{R}_d(\tau_0) - \underset{\sim}{I}(t_n)\,\underset{\sim}{o}^H(t_n) \right] , \tag{48}$$

converging to

$$\underset{\sim}{W} = \underset{\sim}{R}_i^{-1}\,\underset{\sim}{R}_d(\tau_0) . \tag{49}$$

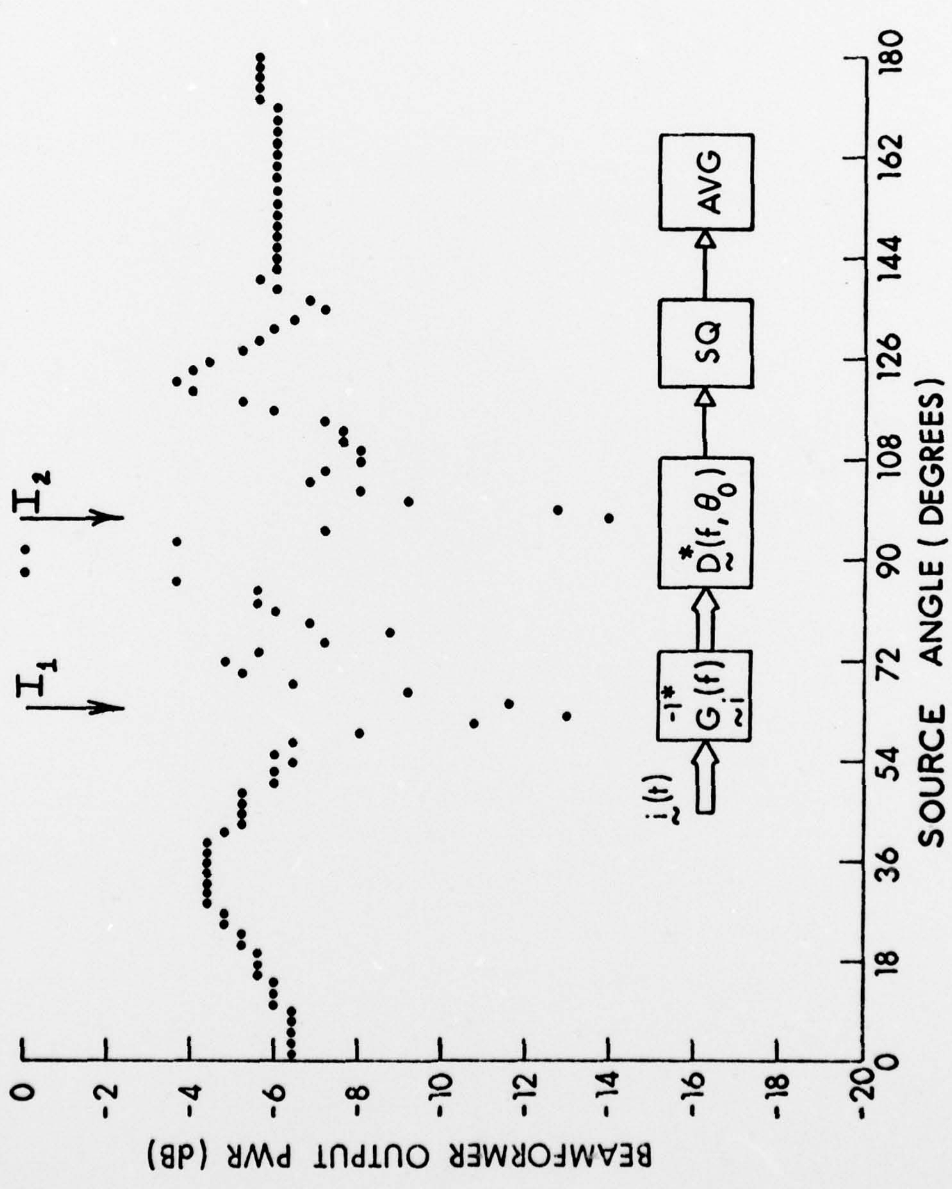
4. SOME SIMULATION RESULTS ADAPTING THE TDL WEIGHTS

The algorithm for obtaining the TDL weights has been programmed on a digital computer using a simulated noise field consisting of two plane wave interferences and white independent noise. The bandpass spectra of the interferences were determined by 6-pole recursive digital filters and had a flat characteristic with bandwidths equal to 1.2 relative to the center frequencies. Each interference was taken to have the same center frequency. For the initial results presented here a very sparse geometrically spaced 5-element array was used. The spacing between the closest elements was 1.06 λ_0 and the geometric factor was 1.5. Ten weights per channel with a tap spacing of $\Delta = 1/4f_0$ were used.

The weights were updated at time instants separated by $1/10f_0$ and were found to converge in a time comparable to that required for LMS beamformer weights obtained via the Griffiths algorithm. The following broadband beam pattern illustrates the performance against 0 dB power interferences at 68^o and 95^o, with -10 db white noise power.

The beam pattern was accomplished by fixing the weights, beamforming in the prescribed direction and sweeping a single plane wave signal around in source angle. The reduced response in the direction of the interferences illustrates the desirable properties of this system.

ADAPTIVE MULTIBEAM SYSTEM RESPONSE FOR LOOK ANGLE θ_0=90°



5. CONCLUSIONS

The two algorithms derived in this paper generalize an existing adaptive filtering method to include discrete paprameter time or frequency domain realization of the class of filters which give prescribed input-output second order statistics. One important application that is investigated is the realization of the prebeamformer matrix filter used by a large class of optimum multibeam systems. The two algorithms are shown to iteratively converge to the desired solutions and to have noise fluctuations on the steady state weights which can be made arbitrarily small by proper choice of the scaling parameter. The time domain TDL adaptive algorithms has been simulated and shown to perform as predicted. However, a good deal of experimental investigation of the performance of both algorithms under a variety of noise field conditions remains to be done.

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REFERENCES

1. H. L. Van Trees, "A Unified Theory for Optimum Array Processing," Rpt. No. 4160866, A.D. Little, Inc., Cambridge, Mass., Aug. 1966.

2. A. H. Nuttall and D. W. Hyde, "A Unified Approach to Optimum and Sub-Optimum Processing with Arrays," Rpt. No. 992, Navy Underwater Sound Laboratory, April, 1969.

3. H. Cox, "Optimum Arrays and the Schwartz Inequality," JASA, Vol. 45, No. 1, pp. 233-244, Jan. 1969.

4. L. Griffiths, "Signal Extraction Using Real-Time Adaptation of Linear Multi-channel Filter," Tech. Rpt. No. 6788-1, Systems Theory Lab., Stanford University, Feb. 1968.

REFERENCES (cont'd.)

5. T. P. Daniell, "Adaptive Estimation with Mutually Correlated Training Samples," Tech. Rpt. No. 6778-4, Systems Theory Lab., Stanford University, Aug. 1968.

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